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Effects of variation in root temperature on heat lost from a thermally non-symmetric fin

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Abstract—A discussion of the variation of the heat lost from a fin is presented the root temperature of which is $T = T_w + a\cos^m(\pi y'/2l)$, $0 \leqslant m \leqslant 5$, and surface convection coefficients of which are constant but unequal. The range of parameters are $0 \leqslant B_2 \leqslant B_1 \leqslant 1$, $B_1 = 0.01, 0.1, 1.0$ where B_1 and B_2 are the Biot numbers of the top and bottom surfaces, respectively. A fin justification criterion (fin heat loss greater than or equal to three times the no-fin heat loss) is used to discuss fin effectiveness. The results are (1) the heat lost from the fin decreases as m increases, $a > 0$; (2) tip Biot number is not an important factor in heat lost for large B_1 and B_2 and (3) as a increases (m constant) the heat lost increases almost linearly.

INTRODUCTION

A COMMON simplification made when analyzing an extended surface is that the temperature variation within the fin is considered to be one-dimensional. Many papers [1–7] have shown this one-dimensional approach, although convenient, may cause error under certain physical conditions (e.g. when the convection coefficient, h, is large compared to the fin material thermal conductivity). Typically the validity criterion is described by the root Biot number magnitude; for constant h, the one-dimensional assumption is that the Biot number, based on the half thickness of the fin, must be less than 0.1. Another situation where the usual one-dimensional assumption may be in error is when the convection coefficients of the top and bottom surfaces are not equal. Finally, the actual base temperature of the fin is not really constant as usually assumed in many analytical works [8–11].

The effort of the study reported here was directed at determining the variation of the heat lost from a fin for a non-constant root temperature, $T = T_w + a\cos^m(\pi y'/2l)$, $m = 0$–5, when the convection coefficients of all surfaces are constant but not equal and thermal radiation effects are neglected. So the purpose of this paper is to provide insight into the effects of non-constant root temperature and the effect of unequal top, bottom and tip surface convection coefficients on the heat lost from a fin of rectangular profile.

The analysis is based upon the usual assumptions [12]:

(1) The thermophysical properties, the heat transfer and the temperature distribution are independent of time.
(2) The fin material is homogeneous and isotropic.
(3) There are no heat sources within the fin.
(4) Newton's law of cooling is valid.

In describing the convection characteristics, the Biot number, $B(=hl/k)$, will be used rather than the convection coefficient, with the restrictions being (1) $0 \leqslant B_2 \leqslant B_1 \leqslant 1$; (2) $B_1 = 0.01$, 0.1, 1.0; (3) $0 \leqslant B_1 \leqslant 1.0$ and (4) the non-constant root temperature variation factor $b = a/\theta_0$ varies from -0.5 to 0.5. Note, the subscripts of the Biot numbers, 1, 2 and 3, denote the top, bottom, and tip surfaces of the fin, respectively. For the quantitative results of this study, the length to one-half the root dimension of the fin was arbitrarily selected to be 5 and the heat lost from the fin is denoted by a non-dimensional form, $Q/k\theta_0$.

TWO-DIMENSIONAL ANALYSIS

In the case of a two-dimensional rectangular fin and constant physical properties, the equation which describes the temperature profile, deduced from the first law of thermodynamics, is

$$\frac{\partial^2 T}{\partial x'^2} + \frac{\partial^2 T}{\partial y'^2} = 0. \tag{1}$$

The boundary conditions for the fin illustrated in Fig. 1 are

$$\left.\begin{aligned} &x' = 0, T = T_w + a\cos^m\left(\frac{\pi y'}{2l}\right) && (2)\\ &x' = L', -k\frac{\partial T}{\partial x'} = h_3(T - T_\infty) && (3)\end{aligned}\right\} -l \leqslant y' \leqslant l$$

$$\left.\begin{aligned} &y' = l', -k\frac{\partial T}{\partial Y'} = h_1(T - T_\infty) && (4)\\ &y' = -l, k\frac{\partial T}{\partial y'} = h_2(T - T_\infty) && (5)\end{aligned}\right\} 0 \leqslant x' \leqslant L'.$$

Note from Fig. 2 the versatility of the root boundary

NOMENCLATURE

a	maximum root depression (elevation) temperature
b	a/θ_0
m	exponent of the non-constant root temperature factor
B_1	fin top surface Biot number, h_1l/k
B_2	fin bottom surface Biot number, h_2l/k
B_3	fin tip surface Biot number, h_3l/k
B_w	root surface Biot number, h_wl/k
h_1	fin top surface convection coefficient
h_2	fin bottom surface convection coefficient
h_3	fin tip surface convection coefficient
h_w	root surface convection coefficient
k	thermal conductivity
l	one half fin thickness
L'	fin length
L	L'/l
Q	heat loss by the fin per length along the root under steady-state condition [W m^{-1}]
T	fin temperature
T_w	fin root temperature
T_∞	ambient temperature
x'	along the fin variable (root to tip), $\leqslant L$
x	x'/l
y'	across the fin variable, $\leqslant \lvert l \rvert$
y	y'/L.

Greek symbols

θ	adjusted fin temperature excess, $(T-T_\infty)$
θ_0	T_w-T_∞
λ_n	eigenvalue.

condition in representing various symmetric root temperature conditions.

If we let $\theta = T-T_\infty$, $\theta_0 = T_w-T_\infty$, $L = L'/l$, $x = x'/l$ and $y = y'/l$ then equation (1) assumes the form

$$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2} = 0 \tag{6}$$

while equations (2)–(5) transform to

$$x = 0, \theta = \theta_0 + a\cos^m\left(\frac{\pi y}{2}\right) \tag{7}$$

$$x = L, \frac{\partial\theta}{\partial x}+B_3\theta = 0 \tag{8}$$

$$\left.\right\} -1 \leqslant y \leqslant 1$$

$$y = 1, \frac{\partial\theta}{\partial y}+B_1\theta = 0 \tag{9}$$

$$y = -1, \frac{\partial\theta}{\partial y}-B_2\theta = 0 \tag{10}$$

$$\left.\right\} 0 \leqslant x \leqslant L$$

where $B_i = h_il/k$, $i = 1, 2, 3$ and $m = 0, 1, \ldots, 5$.

Using the separation of variables procedure, the solution of equation (6) is found to be

$$\theta = \theta_0 \sum_{n=1}^{\infty} f_1(y)f_2(x)N_{nm} \tag{11}$$

where

$$f_1(y) = \cos(\lambda_n y)+A_n \sin(\lambda_n y) \tag{12}$$

$$f_2(x) = \cosh(\lambda_n x)+f_n \sinh(\lambda_n x) \tag{13}$$

$$f_n = -\frac{B_3+\lambda_n \tanh(\lambda_n L)}{\lambda_n+B_3 \tanh(\lambda_n L)} \tag{14}$$

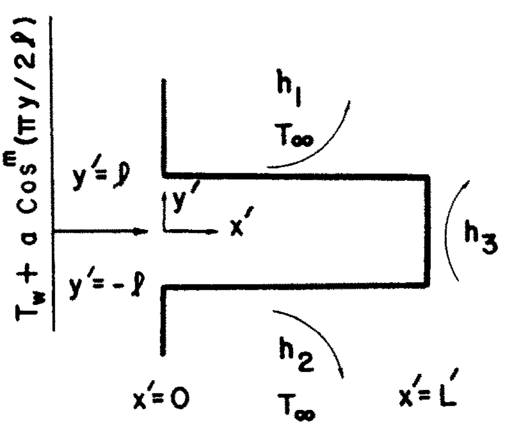

FIG. 1. Geometry of a thermally asymmetric, constant cross-sectional area, rectangular fin.

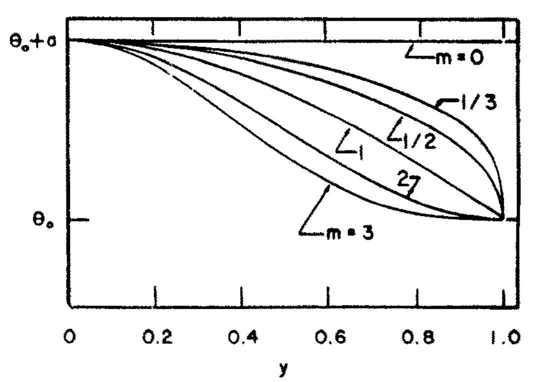

FIG. 2. Demonstration of the root temperature variation dependence upon the parameter m.

Effects of

$$D_n(1+b) \tag{15}$$

$$D_n\left[1+\frac{2b\pi\lambda_n}{\pi^2-4\lambda_n^2}\cot(\lambda_n)\right] \tag{16}$$

$$N_{n2} = D_n\left[1+\frac{b\pi^2}{2(\pi^2-\lambda_n^2)}\right] \tag{17}$$

$$N_{n3} = D_n\left[1+\frac{12\pi^3 b\lambda_n\cot(\lambda_n)}{(\pi^2-4\lambda_n^2)(9\pi^2-4\lambda_n^2)}\right] \tag{18}$$

$$N_{n4} = D_n\left[1+\frac{3\pi^4 b}{2(4\pi^2-\lambda_n^2)(\pi^2-\lambda_n^2)}\right] \tag{19}$$

$$N_{n5} = D_n\left[1+\frac{240\pi^5\lambda_n b\cot(\lambda_n)}{(\pi^2-4\lambda_n^2)(9\pi^2-4\lambda_n^2)(25\pi^2-4\lambda_n^2)}\right] \tag{20}$$

$$N_{nm} = D_n\left[1+\frac{b\lambda_n\Gamma(m+1)}{2^m\sin(\lambda_n)\Gamma\left(\frac{m+2}{2}+\frac{\lambda_n}{\pi}\right)\Gamma\left(\frac{m+2}{2}-\frac{\lambda_n}{\pi}\right)}\right] \tag{21}$$

$$D_n = \frac{\frac{2\sin(\lambda_n)}{\lambda_n}}{\left[\left(1+\frac{\sin(2\lambda_n)}{\lambda_n}\right)+A_n^2\left(1-\frac{\sin(2\lambda_n)}{\lambda_n}\right)\right]} \tag{22}$$

$$A_n = \frac{\lambda_n\tan(\lambda_n)-B_1}{\lambda_n+B_1\tan(\lambda_n)} = \frac{-\lambda_n\tan(\lambda_n)+B_2}{\lambda_n+B_2\tan(\lambda_n)} \tag{23}$$

and

$$b = \frac{a}{\theta_0}.$$

The values of λ_n were obtained from the two right-hand portions of equation (23) using a Newton–Raphson method.

The heat lost per fin width in this two-dimensional case is

$$\begin{aligned} Q &= \int_{-l}^{l}\left[-k\frac{\partial T}{\partial x'}\right]_{x'=0}\mathrm{d}y' \\ &= \int_{-1}^{1}\left[-k\frac{\partial\theta}{\partial x}\right]_{x=0}\mathrm{d}y \\ &= -2k\theta_0\sum_{n=1}^{\infty}\sin(\lambda_n)f_n N_{nm}. \end{aligned} \tag{24}$$

In determining the usefulness of a fin, comparisons are usually made with the 'no-fin' or 'bare wall' condition. Thus, if no fin were present and the bare wall between $-l$ and l exhibited the same root temperature profile, the heat lost per fin would be

$$\begin{aligned} Q(\text{no fin}) &= \int_{-l}^{l} h_w(T-T_\infty)\,\mathrm{d}y' \\ &= kB_w\theta_0\left[2+b\int_{-1}^{1}\cos^m\left(\frac{\pi y}{2}\right)\mathrm{d}y\right]. \end{aligned} \tag{25}$$

Comparisons between equations (24) and (25) may be used in an effort to determine the usefulness of a fin. For example, a criterion for a fin to be justified might be that

$$\frac{Q}{Q(\text{no fin})} \geqslant 3. \tag{26}$$

RESULTS

For eigenvalues, λ_n, equation (23) was used for given B_1 and B_2. When B_1 is not equal to B_2, the solution to this eigenfunction equation is complicated. In order to demonstrate this, consider, for example, that for $\lambda_n < 10$, there are four eigenvalues in the case of $B_1 = B_2$ while there are seven eigenvalues in the case of $B_1 = 1$ and $B_2 = 0$. This is illustrated in Figs. 3 and 4. In both of these figures $f(\lambda_n)$ is the difference between the two right-hand portions of equation (23).

A form of the solution is represented in Fig. 5. Presented in this figure is the variation of the heat lost from a thermally asymmetric fin for values of m ranging from 0 to 5, B_2/B_1 ranging from 0 to 1 ($B_1 = 0.01$, 0.1, 1.0), $b = 0.5$ and $B_3 = 0$. $B_3 = 0$ represents the insulated fin tip case. In all cases, the heat lost from a fin and the slope of the heat lost curve with respect to m decreases as m increases for all B_1. Results for the same conditions as in Fig. 5 except that the fin tip is not insulated (i.e. $B_3 = 1$) are depicted in Fig. 6. When $B_1 = 1$, the effect of B_3 on the heat lost is very small. This can be seen by comparing Figs. 5 and 6 and noting that there is very little difference between the two sets of $B_1 = 1$ curves. The heat lost from the fin increases a great deal for $B_1 = 0.01$ when this comparison is made. When we further compare Figs. 5 and 6, another characteristic feature becomes apparent; the difference in the heat lost with respect to the change of B_2/B_1 becomes small as B_3 changes from 0 to 1 for small B_1. For example, in the case of $m = 1$, the values of $Q(B_2/B_1 = 1)/Q(B_2/B_1 = 0)$ are 1.929 at $B_1 = 0.01$, 1.639 at $B_1 = 0.1$ and 1.506 at $B_1 = 1$, for $B_3 = 0$ while the values $Q(B_2/B_1 = 1)/Q(B_2/B_1 = 0)$ are 1.053 at $B_1 = 0.01$, 1.286 at $B_1 = 0.1$ and 1.49 at $B_1 = 1$ for $B_3 = 1$. From Table 1, we can also see that $Q(B_2/B_1 = 1)/Q(B_2/B_1 = 0)$ increases as the value of B_1 decreases when $B_3 = 0$ but it decreases as the value of B_1 decreases when $B_3 = 1$. It is interesting to note that the change of B_3 does not affect the slope of the heat lost curve with respect to m as B_3 changes from 0 to 1 when $B_1 = 1$. The slope does begin to change slightly for low values of m when $B_1 = 0.1$, and the slope becomes noticeably different when $B_1 = 0.01$ as B_3 changes from 0 to 1.

Superimposed on both Figs. 5 and 6 are the results

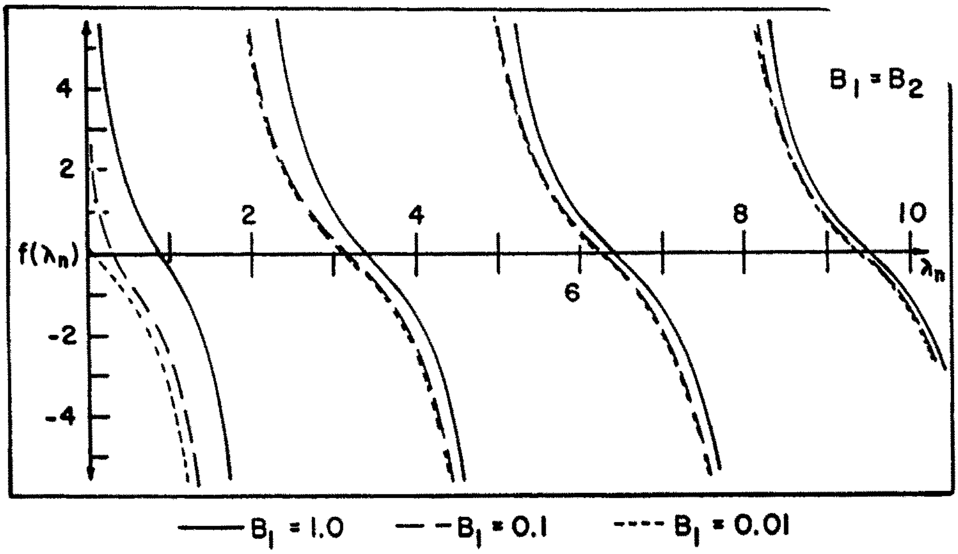

FIG. 3. The eigenvalues when the top surface Biot number is equal to the bottom surface Biot number.

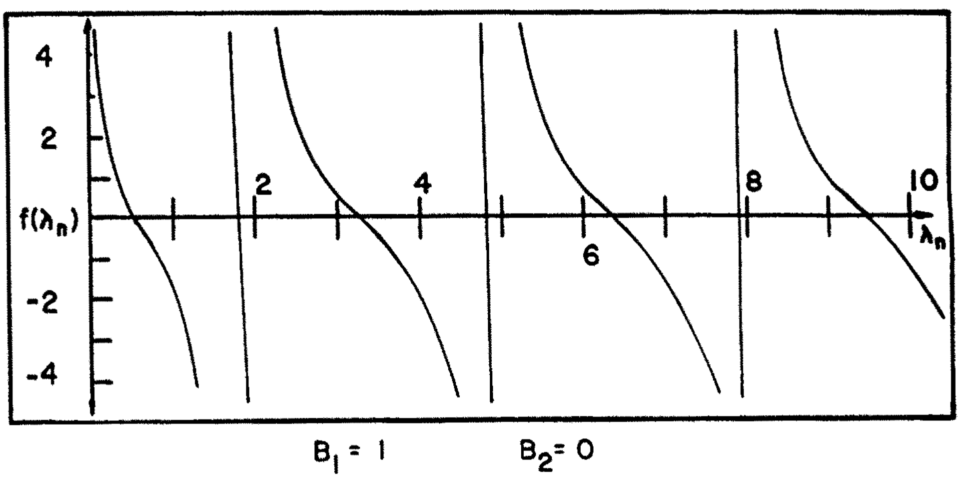

FIG. 4. The eigenvalues when the top surface Biot number, B_1, is not equal to the bottom surface Biot number, B_2.

of applying the suggested justification criterion. In order to apply the criterion, the results of Table 2 may be used. That is, Table 2(a) is a listing illustrating equation (25) for various values of m (note the entries in this table are proportional to the average of the difference in the root temperature and the ambient temperature, T_∞). Table 2(b) presents the special cases of $b = 0.5$ and $B_w = B_3 = 0.01$, 0.1 and 1.0. Note that Table 2 indicates that when $(b > 0)$ the result is a reduced heat loss as m increases. For the points on Figs. 5 and 6, $B_w = 0.01$ and 0.1. The results for $B_w = 1$ are not included because of their magnitude. Although the insulated tip example is not realistic, a comparison between the fin values and three times the no-fin values indicate that fins are justified for $B_1 < 0.1$ and that for B_1 near 0.1, larger values of B_2 are required. Further as the magnitude of B_1 and thus B_2 decreases, even a fin with an insulated lower surface is justified. The other physical extreme $(B_3 = 1)$ is presented in Fig. 6 and results in a slight relaxing of the range of values of B_2 for $B_1 = 0.1$ for justification by the criterion. Note also that the heat lost in the $B_1 = 0.01$ case is almost 20 times that of the no-fin case.

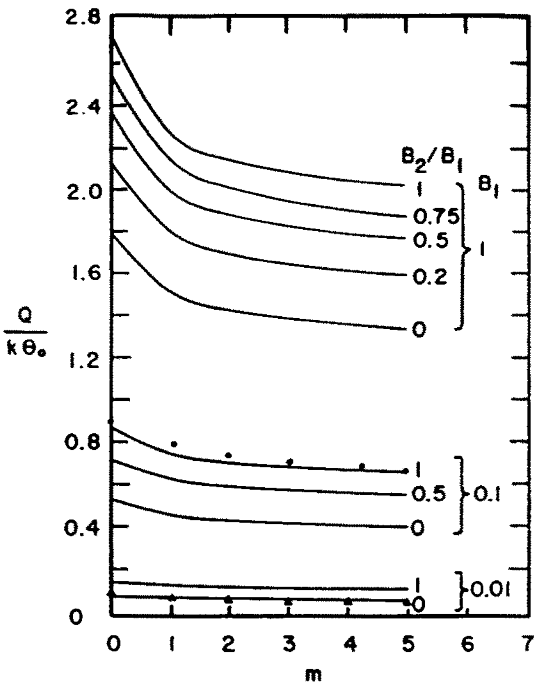

FIG. 5. The non-dimensional heat lost from a fin for $0 \leq m \leq 5$, $0 \leq B_2/B_1 \leq 1$, $b = 0.5$, $B_3 = 0$ and $B_1 = 0.01$, 0.1, 1.0. Included are three times of the no-fin heat loss value for $B_w = 0.1$(●) and 0.01(▲).

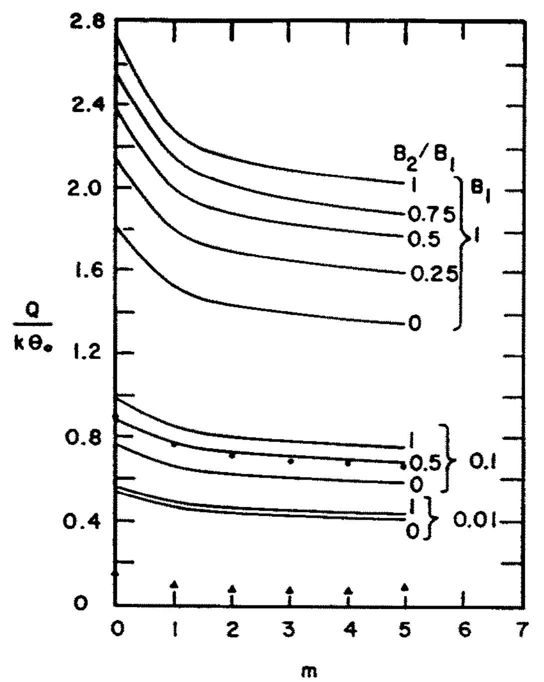

FIG. 6. The non-dimensional heat lost from a fin for $0 \leq m \leq 5$, $0 \leq B_2/B_1 \leq 1$, $b = 0.5$, $B_3 = 1$ and $B_1 = 0.01$, 0.1, 1.0. Included are three times of the no-fin heat loss value for $B_w = 0.1$(●) and 0.01(▲).

The variation of the heat lost from a fin as the value of b varies from -0.5 to 0.5 and m varies from 0 to 5 when $B_2/B_1 = 0.5$ and $B_3 = 0$ in the cases of $B_1 = 0.01$, 0.1 and 1.0 is illustrated in Fig. 7. From Fig. 7, it can be seen that the heat lost from a fin varies almost linearly with b for all m and the slope decreases as m increases. Further, we are lead to believe that the slope may approach zero as m approaches infinity. This, of course, is not unexpected since as m increases the highest temperature in the root of the fin is near the center (i.e. buried or encapsulated deep in the fin). Thus the decrease (increase) in the heat lost as m increases for a given $b > 0$ ($b < 0$) should be expected. In fact, as m approaches infinity, the average root temperature is decreased so much that there is constant heat transfer in the limit. Note further that, for example, when $b = 0.5$, the values of $(Q(m=1) - Q(m=5))/Q(m=1)$ are 0.1124 at $B_1 = 0.01$, 0.1125 at $B_1 = 0.1$ and 0.1107 at $B_1 = 1$. When $b = 0.25$, the values of $(Q(m=1) - Q(m=5))/Q(m=1)$ are 0.0654 at $B_1 = 0.01$, 0.0640 at $B_1 = 0.1$ and 0.0621 at $B_1 = 1$. From these data and the fact that the slope varies almost linearly, the ratio $(Q(m=1) - Q(m=5))/bQ(m=1)$ is essentially constant for different values of B_1 but for the same values of B_2/B_1. Finally, as expected, in all examples for $b > 0$, the $m = 0$ case produces the best heat transfer conditions.

The effect of B_3 on the heat lost from a horizontal fin for values of m from 0 to 5 when $b = 0.5$ and $B_2/B_1 = 0.5$, in the cases of $B_1 = 0.01$, 0.1, 1.0 is represented in Fig. 8. As we can see from this figure, when $B_1 = 1$, B_2 has a negligible effect on the heat lost for all m. But the effect of B_3 becomes larger as the value of B_1 decreases particularly for small B_3. For $m = 1$ and $B_1 = 1$, the heat lost from the fin increases by

Table 1. Comparison of the ratio, $Q(B_2/B_1 = 1)/Q(B_2/B_1 = 0)$, with respect to m and B_1 when $B_3 = 0$ and 1

		$Q(B_2/B_1 = 1)/Q(B_2/B_1 = 0)$ $B_3 = 0$	$B_3 = 1$
$m = 0$	$B_1 = 0.01$	1.931	1.055
	$B_1 = 0.1$	1.648	1.295
	$B_1 = 1$	1.524	1.509
$m = 1$	$B_1 = 0.01$	1.929	1.053
	$B_1 = 0.1$	1.639	1.286
	$B_1 = 1$	1.506	1.490
$m = 2$	$B_1 = 0.01$	1.931	1.053
	$B_1 = 0.1$	1.642	1.287
	$B_1 = 1$	1.508	1.488
$m = 3$	$B_1 = 0.01$	1.933	1.053
	$B_1 = 0.1$	1.645	1.289
	$B_1 = 1$	1.509	1.494
$m = 4$	$B_1 = 0.01$	1.934	1.053
	$B_1 = 0.1$	1.647	1.290
	$B_1 = 1$	1.510	1.495
$m = 5$	$B_1 = 0.01$	1.935	1.053
	$B_1 = 0.1$	1.648	1.291
	$B_1 = 1$	1.511	1.496

Table 2. Variation of $Q(\text{no fin})/B_w k\theta_0$ vs m

(a) For general parameter b and B_w

m	0	1/2	1	2	3	4	5
$Q(\text{no fin})/B_w k\theta_0$	$2(1+b)$	$2(1+0.76276b)$	$2(1+2b/\pi)$	$2(1+b/2)$	$2(1+4b/3\pi)$	$2(1+3b/8)$	$2(1+16b/15\pi)$

(b) For the cases of $b = 0.5$ and $B_w = 0.01$, 0.1 and 1.0

	m	0	1/2	1	2	3	4	5
$Q(\text{no fin})/k\theta_0$	$B_w = 0.01$	0.03	0.028	0.026	0.025	0.024	0.024	0.023
	$B_w = 0.1$	0.3	0.276	0.264	0.250	0.242	0.238	0.234
	$B_w = 1$	3	2.763	2.637	2.500	2.424	2.375	2.339

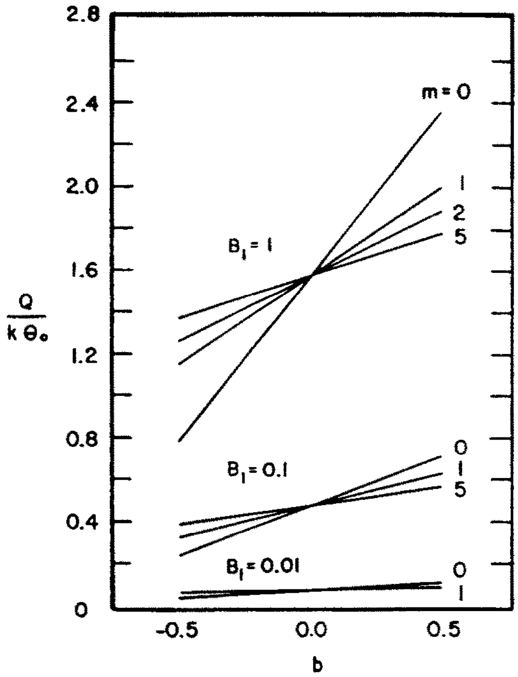

FIG. 7. The non-dimensional heat lost from a fin for $-0.5 \leqslant b = a/\theta_0 \leqslant 0.5$ and $0 \leqslant m \leqslant 5$ when $B_2/B_1 = 0.5$ and $B_3 = 0$.

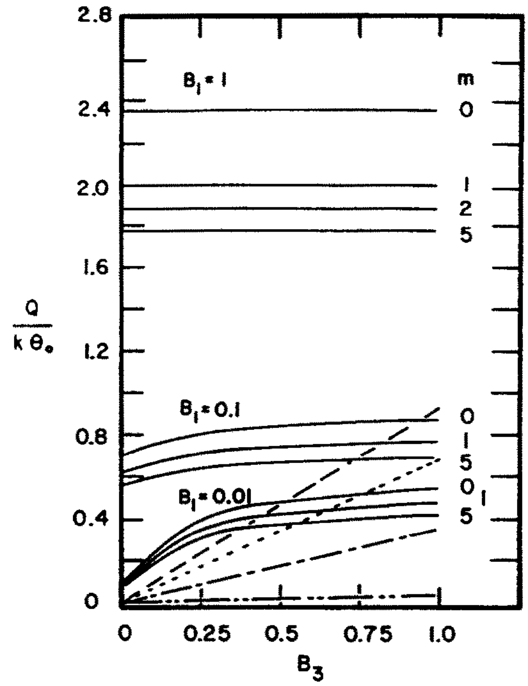

FIG. 8. The effect of B_3 on the non-dimensional heat lost from a fin for $0 \leqslant m \leqslant 5$ when $b = 0.5$ and $B_2/B_1 = 0.5$, for the cases of $B_1 = 1$, 0.1, 0.01. Included are the three times of the no fin heat loss values for $B_w = B_3$ and $m = 0$, $B_1 = 0.1$ ( ), $m = 5$, $B_1 = 0.1$ (···), $m = 0$, $B_1 = 0.01$ (· ·) and $m = 5$, $B_1 = 0.01$ (·· ··).

only a factor of 1.0012 as B_3 varies from 0 to 1, while it increases by 1.237 as B_3 varies from 0 to 1 at $B_1 = 0.1$. The most dramatic increase occurs for $B_1 = 0.01$; the increase is by a factor of 5.2 as B_3 varies from 0 to 1. In particular, for $B_1 = 0.01$ its increase factor is 3.75 as B_3 varies from 0 to 0.25 and increases by only 1.05 as B_3 varies from 0.75 to 1. So, at small B_1, even though the effect of B_3 on the heat lost is important, its effect decreases as B_3 increases. These effects for B_3 are also expected. That is, in either the case of decreased top and/or bottom heat transfer ($B_i \to$ small numbers, $i = 1$, 2) or encapsulated high temperature areas ($m \to$ large numbers), the effects of B_3 would become more influential. Finally, included in this figure is the suggested justification criterion for several cases. Note that for this figure, $B_w = B_3$.

CONCLUSION

The results presented produce the following straightforward conclusions ($b > 0$):

(1) The non-dimensional heat transfer ($Q/k\theta_0$) depends in varying degrees upon, B_1, B_2, B_3, L, b and m.

(2) The heat lost from a horizontal fin decreases as m increases.

(3) The change of the value of B_3 does not affect the slope of the heat lost curve for large B_1 ($B_1 = 1$).

(4) $Q(B_2/B_1 = 1)/Q(B_2/B_1 = 0)$ increases as the value of B_1 decreases when $B_3 = 0$ but it decreases as the value of B_1 decreases when $B_3 = 1$.

(5) For all m, the heat lost from a fin varies almost linearly with b and the slopes become smaller as m increases.

(6) When the value of B_1 is small, the effect of fin tip convection coefficient on the heat lost from a fin is very important but its effect remarkably decreases as the value of B_3 increases.

From a fin designer's standpoint, no drastic changes in the conditions for justification for the addition of a fin exists even for this two-dimensional situation. The top surface Biot number being less than 0.1, for most values of the bottom surface and tip Biot numbers, appears to be a valid standard as long as the root temperature variation is not too dramatic.

REFERENCES

1. M. Avrami and J. B. Little, Diffusion of heat through a rectangular bar and the cooling and insulating effect of fins, *J. Appl. Phys.* **13,** 255–264 (1942).
2. L. C. Burmeister, Triangular fin performance by the heat balance integral method, *ASME J. Heat Transfer* **101,** 562–564 (1979).
3. R. K. Irey, Errors in the one-dimensional fin solution, *ASME J. Heat Transfer* **90,** 175–176 (1968).
4. H. H. Keller and E. V. Somers, Heat transfer from an annular fin of constant thickness, *ASME J. Heat Transfer* **81,** 151–156 (1959).
5. W. Lau and C. W. Tan, Errors in one-dimensional heat transfer analyses in straight and annular fins, *ASME J. Heat Transfer* **95,** 549–551 (1963).
6. A. D. Snider and A. D. Kraus, Recent developments in the analyses and design of extended surfaces, *ASME J. Heat Transfer* **105,** 302–306 (1983).
7. H. C. Unal, The effect of the boundary condition at a fin tip on the performance of the fin with and without internal heat generation, *Int. J. Heat Mass Transfer* **31,** 1483–1496 (1988).
8. E. M. Sparrow and D. K. Hennecke, Temperature depression at the base of a fin, *ASME J. Heat Transfer* **92,** 204–206 (1970).
9. E. M. Sparrow and L. Lee, Effects of fin base-temperature depression in a multifin array, *ASME J. Heat Transfer* **97,** 463–465 (1975).
10. D. C. Look, Jr., 2-D fin performance: *Bi*(top) > *Bi*(bottom), *ASME J. Heat Transfer* **111,** 780–782 (1988).
11. D. C. Look, Jr., Two-dimensional fin with non-constant root temperature, *Int. J. Heat Mass Transfer* **32,** 977 (1989).
12. A. D. Kraus, Analysis of extended surface, *ASME J. Heat Transfer* **110,** 1071–1073 (1988).

EFFET DE LA VARIATION DE LA TEMPERATURE DE BASE SUR LA PERTE THERMIQUE D'UNE AILETTE THERMIQUEMENT NON SYMETRIQUE

Résumé—On présente une discussion de la variation de la perte thermique d'une ailette dont la température de base est $T = T_w + a\cos^m(\pi y'/2l)$, $0 \leqslant m \leqslant 5$, et dont les coefficients de convection sont constants mais différents. Le domaine des paramètres sont $0 \leqslant B_2 \leqslant B_1 \leqslant 1$, $B_1 = 0{,}01$; 0,1 ; 1,0, où B_1 et B_2 sont respectivement les nombres de Biot des surfaces au sommet et au pied. Un critère (perte thermique avec ailette supérieure ou égal à trois fois la perte sans ailette) est utilisé pour discuter l'efficacité de l'ailette. Les résutats sont : (1) la perte thermique de l'ailette décroît quand m augmente, $a > 0$; (2) le nombre de Biot au sommet n'est pas un facteur important pour la perte si B_1 et B_2 sont grands ; (3) quand a augmente (m constant) la perte thermique croît à peu près linéairement.

EINFLUSS EINER VERÄNDERLICHEN FUSSTEMPERATUR AUF DIE WÄRMEABGABE EINER THERMISCH NICHT-SYMMETRISCHEN RIPPE

Zusammenfassung—Es werden die Schwankungen der Wärmeabgabe an einer Rippe dargestellt, deren Fußtemperatur gemäß $T = T_w + a\cos^m(\pi y'/2l)$, $0 \leqslant m \leqslant 5$ schwankt, während die Wärmeübergangskoeffizienten bei Konvektion zwar konstant, aber unterschiedlich sind. B_1 und B_2 sind die Biot-Zahlen an den oberen und unteren Oberflächen, sie schwanken innerhalb $0 \leqslant B_2 \leqslant B_1 \leqslant 1$, wobei $B_1 = 0{,}01$; 0,1 und 1,0. Der Rippenwirkungsgrad wird mit Hilfe eines Kriteriums zur Rechtfertigung von Rippen diskutiert: danach soll die Wärmeabgabe mit Rippe wenigstens dreimal so groß sein wie die Wärmeabgabe ohne Rippe. Als Ergebnis wird festgehalten: (1) die Wärmeabgabe von der Rippe nimmt mit wachsendem m ab, $a > 0$; (2) die Biot-Zahl an der Rippenspitze beeinflußt die Wärmeabgabe für große Werte von B_1, B_2 nur gering; (3) die Wärmeabgabe nimmt für konstantgehaltenes m mit wachsendem a fast linear zu.

ВЛИЯНИЕ ИЗМЕНЕНИЯ ТЕМПЕРАТУРЫ У ОСНОВАНИЯ НА ТЕПЛОПОТЕРИ ТЕРМИЧЕСКИ НЕСИММЕТРИЧНОГО РЕБРА

Аннотация—Обсуждается изменение теплопотерь ребра, температура у основания которого составляет $T = T_w + a\cos^m(\pi y'/2l)$, $0 \leqslant m \leqslant 5$, а коэффициенты теплообмена являются постоянными, но неравными. Исследуемые параметры изменяются в диапазонах $0 \leqslant B_2 \leqslant B_1 \leqslant 1$, $B_1 = 0{,}01$; 0,1; 1,0, где B_1 и B_2—числа Био соответственно верхней и нижней поверхностей. Критерий обоснования выбора типа ребра (теплопотери ребра в три или три с лишним раза больше, чем в случае без ребер) используется для анализа его эффективности. Получены следующие результаты: (1) теплопотери ребра уменьшаются с ростом значения m, $a > 0$; (2) число Био у вершины ребра не оказывает существенного влияния на теплопотери при больших значениях B_1 и B_2 и (3) с увеличением значения a (при постоянном значении m) теплопотери возрастают почти линейно.